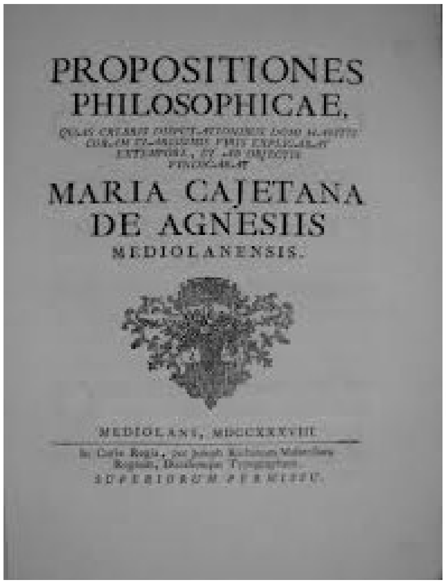
Maria Gaetana Agnesi

(1718-1799)

[Public domain, https://en.wikipedia.org/wiki/Maria_Gaetana_Agnesi#/media/File:Maria_Gaetana_Agnesi.jpg]

Maria Gaetana Agnesi was born in 1718 in Milan, which is now in Italy, but at the time, was part of the Habsburg Empire. She was the eldest of 21 children of Pietro Agnesi (had 3 wives), who came from a wealthy family and was able to afford the best tutors for his daughter. From a young age, Maria demonstrated an amazing memory and intelligence and a proclivity for languages, such as French, Greek, Hebrew, and Latin. As part of Pietro's efforts to achieve a higher social status, he hosted gatherings at the Palazzo Agnesi during which he showed off the intellectual talents of Maria Gaetana and the musical talents of her sister Maria Teresa.

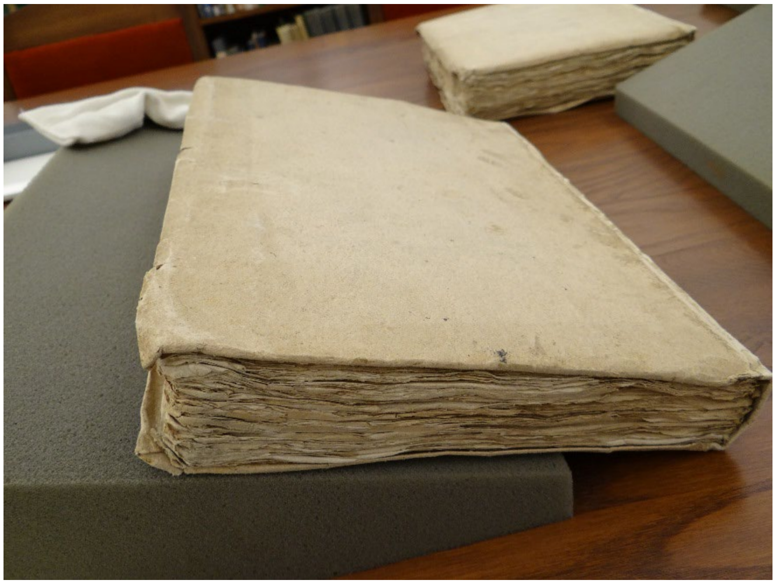
Based on Maria Gaetana's discussions during such gatherings, in 1738, the year she turned 20, she published *Propositiones philosophicae* (*Propositions of Philosophy*), declaring in the prologue "the fitness of women to study the sciences and the arts."



Google Books

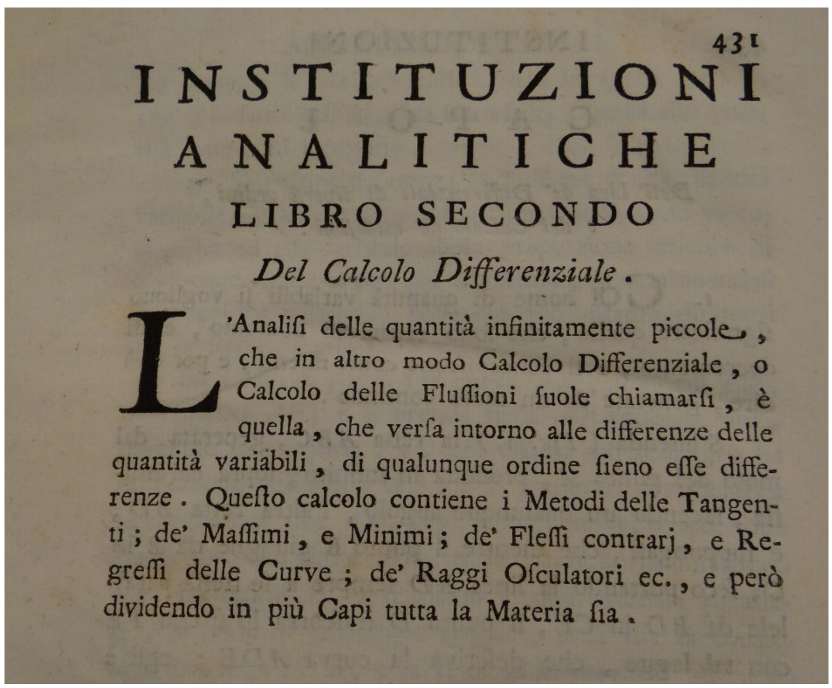
Maria continued her studies of both religion and mathematics, including a non-published commentary on L'Hôpital's *Analysis of Conic Sections*. She explored a variety of writings on calculus including *L' Analyse demontrée*, a 1708 text by French mathematician and priest Charles René Reyneau, aided by Ramiro Rampinelli, a monk, mathematician, and professor. Possibly motivated by having so many younger siblings, Agnesi saw a need for a textbook, which presented mathematics in a natural way and which was written to serve as part of a Christian education for youth.

The result was *Instituzioni Analitiche ad uso della gioventù italiana*, which was published in 1748 in Italian in Milan, a two-volume work on algebra and calculus. The title page is shown below. It is divided into 4 books with Volume 1 containing Book I - *The Analysis of Finite Quantities* and Volume 2 containing Book II – *The Analysis of Quantities Infinitely Small*, Book III – *Of the Integral Calculus*, and Book IV – *The Inverse Method of Tangents*. The book was written as a teaching text; in fact, it was a standard text in Europe for over 100 years, and the work received high praise from a committee of the Académie des Sciences in Paris for uniformly synthesizing and clarifying the work of others on calculus. According to the Dictionary of Scientific Biography, "this book won immediate acclaim in academic circles all over Europe and brought recognition as a mathematician to Agnesi."



You can view a copy of the text at the Linda Hall Library in Kansas City (www.lindahall.org). It is still in its temporary binding as one would have bought it at a printer before taking to a book binder.

The next picture is of the start of Book 2 which is about Differential Calculus. Even though it is in Italian, you can pick out terms like the the analysis of endlessly small quantities or in other terms, Differential Calculus, the Calculus of Fluxions, ….method of tangents, about maxima, minima, ….



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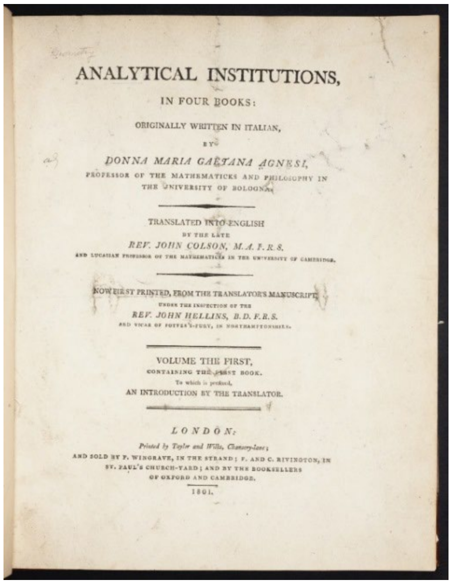
INSTITUZIONI ANALITICHE

LIBRO SECONDO

Del Calcolo Differenziale.

L'Analiſi delle quantità infinitamente piccole, che in altro modo Calcolo Differenziale, o Calcolo delle Fluſſioni ſuole chiamarſi, è quella, che verſa intorno alle differenze delle quantità variabili, di qualunque ordine ſieno eſſe differenze. Queſto calcolo contiene i Metodi delle Tangenti; de' Maſſimi, e Minimi; de' Fleſſi contrarj, e Regreſſi delle Curve; de' Raggi Oſculatori ec., e però dividendo in più Capi tutta la Materia ſia.

The work was translated into English by Rev. John Colson (who learned Italian just so he could translate Analytical Institutions), and the English translation was published in 1801. Here is the title page of the English translation of Volume 1 and the start of a preface by the editor.



ANALYTICAL INSTITUTIONS,

IN FOUR BOOKS:

ORIGINALLY WRITTEN IN ITALIAN,

BY

DONNA MARIA GAETANA AGNESI,

PROFESSOR OF THE MATHEMATICKS AND PHILOSOPHY IN THE UNIVERSITY OF BOLOGNA.

TRANSLATED INTO ENGLISH

BY THE LATE

REV. JOHN COLSON, M.A. F.R.S.

AND LUCASIAN PROFESSOR OF THE MATHEMATICKS IN THE UNIVERSITY OF CAMBRIDGE.

NOW FIRST PRINTED, FROM THE TRANSLATOR'S MANUSCRIPT,

UNDER THE INSPECTION OF THE

REV. JOHN HELLINS, B.D. F.R.S.

AND VICAR OF POTTER'S-PURY, IN NORTHAMPTONSHIRE.

VOLUME THE FIRST,

CONTAINING THE FIRST BOOK.

To which is prefixed,

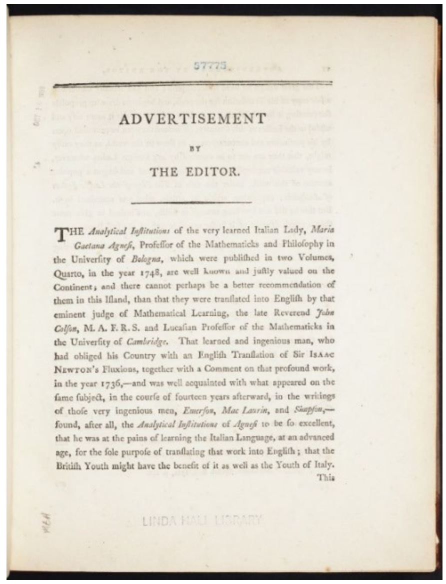
AN INTRODUCTION BY THE TRANSLATOR.

LONDON:

Printed by Taylor and Wilks, Chancery-lane;

AND SOLD BY F. WINGRAVE, IN THE STRAND; F. AND C. RIVINGTON, IN ST. PAUL'S CHURCH-YARD; AND BY THE BOOKSELLERS OF OXFORD AND CAMBRIDGE.

1801.



ADVERTISEMENT

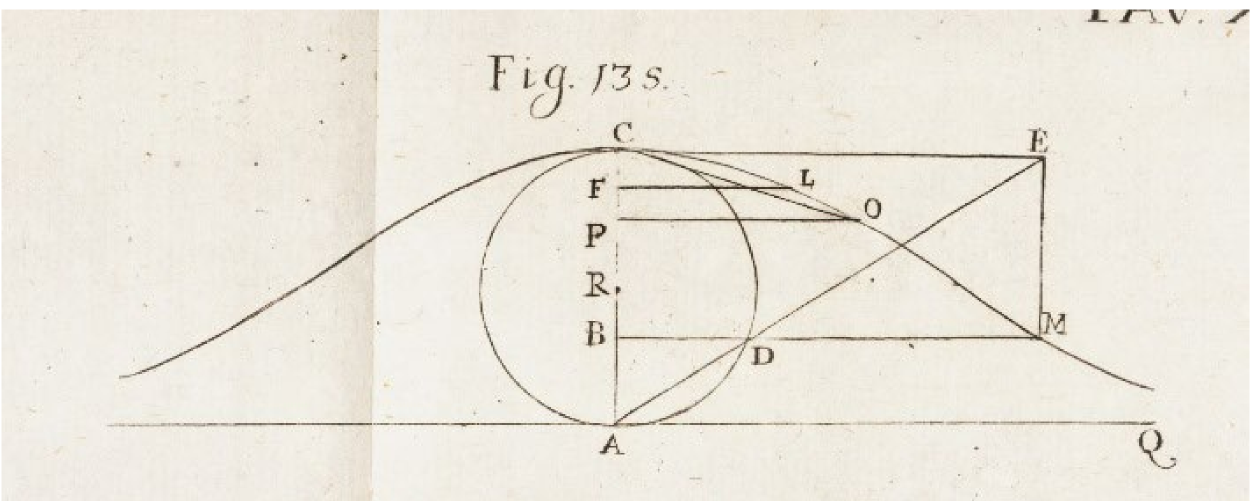
BY

THE EDITOR.

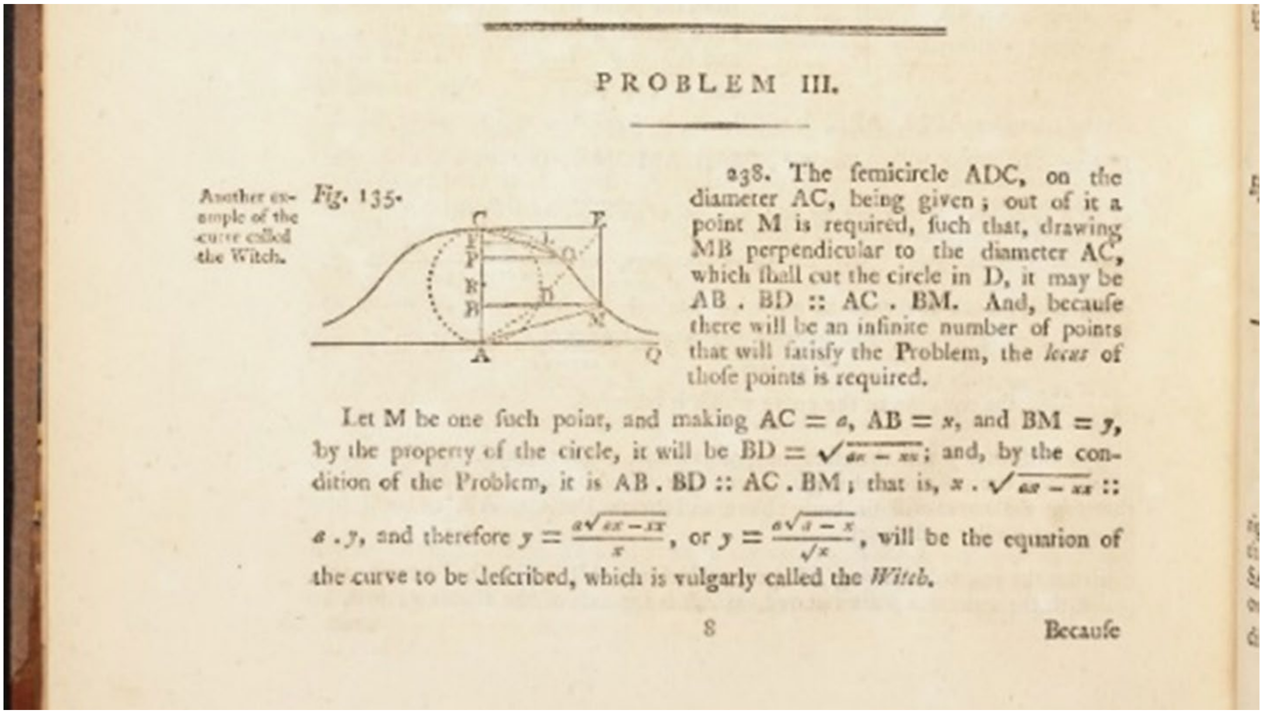
THE *Analytical Inſtitutions* of the very learned Italian Lady, *Maria Gaetana Agneſi*, Profeſſor of the Mathematicks and Philoſophy in the Univerſity of *Bologna*, which were publiſhed in two Volumes, Quarto, in the year 1748, are well known and juſtly valued on the Continent; and there cannot perhaps be a better recommendation of them in this Iſland, than that they were tranſlated into Engliſh by that eminent judge of Mathematical Learning, the late Reverend *John Colſon*, M. A. F. R. S. and Lucaſian Profeſſor of the Mathematicks in the Univerſity of *Cambridge*. That learned and ingenious man, who had obliged his Country with an Engliſh Tranſlation of Sir ISAAC NEWTON's Fluxions, together with a Comment on that profound work, in the year 1736,—and was well acquainted with what appeared on the ſame ſubject, in the courſe of fourteen years afterward, in the writings of thoſe very ingenious men, *Emerſon*, *Mac Laurin*, and *Simpſon*,—found, after all, the *Analytical Inſtitutions* of *Agneſi* to be ſo excellent, that he was at the pains of learning the Italian Language, at an advanced age, for the ſole purpoſe of tranſlating that work into Engliſh; that the Britiſh Youth might have the benefit of it as well as the Youth of Italy.

This

Today many people know of Agnesi due to a mistranslation of the name of a cubic curve she discusses in Analytical Institutions.



While Agnesi called this curve *la Versiera*, Colson translated it as *the Witch* and the curve became known commonly as *the Witch of Agnesi*.



PROBLEM III.

Another example of the curve called the Witch.

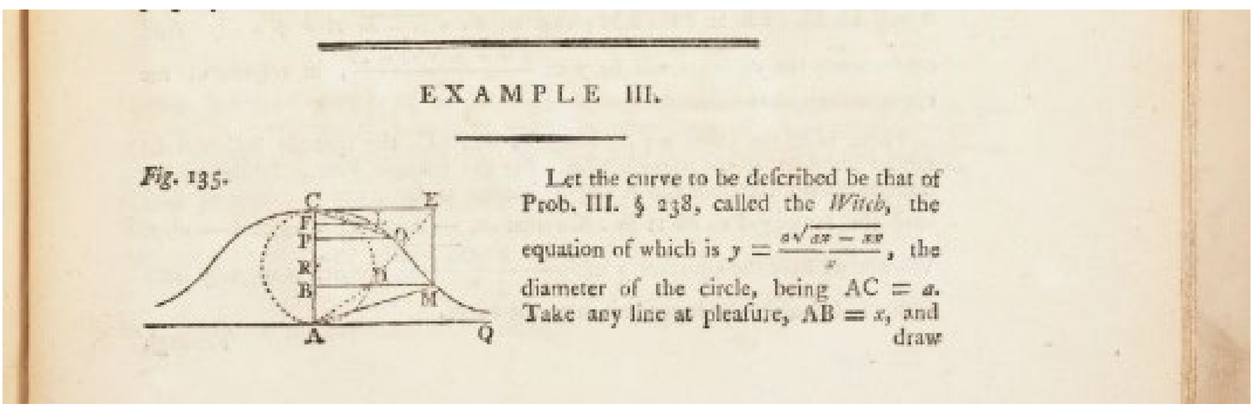
Fig. 135.

238. The ſemicircle ADC, on the diameter AC, being given; out of it a point M is required, ſuch that, drawing MB perpendicular to the diameter AC, which ſhall cut the circle in D, it may be AB . BD :: AC . BM. And, becauſe there will be an infinite number of points that will ſatisfy the Problem, the *locus* of thoſe points is required.

Let M be one ſuch point, and making $AC = a$, $AB = x$, and $BM = y$, by the property of the circle, it will be $BD = \sqrt{ax - xx}$; and, by the condition of the Problem, it is AB . BD :: AC . BM; that is, $x . \sqrt{ax - xx} :: a . y$, and therefore $y = \frac{a\sqrt{ax - xx}}{x}$, or $y = \frac{a\sqrt{a - x}}{\sqrt{x}}$, will be the equation of the curve to be deſcribed, which is vulgarly called the *Witch*.

8 Becauſe

It shows up again later in the book as well.



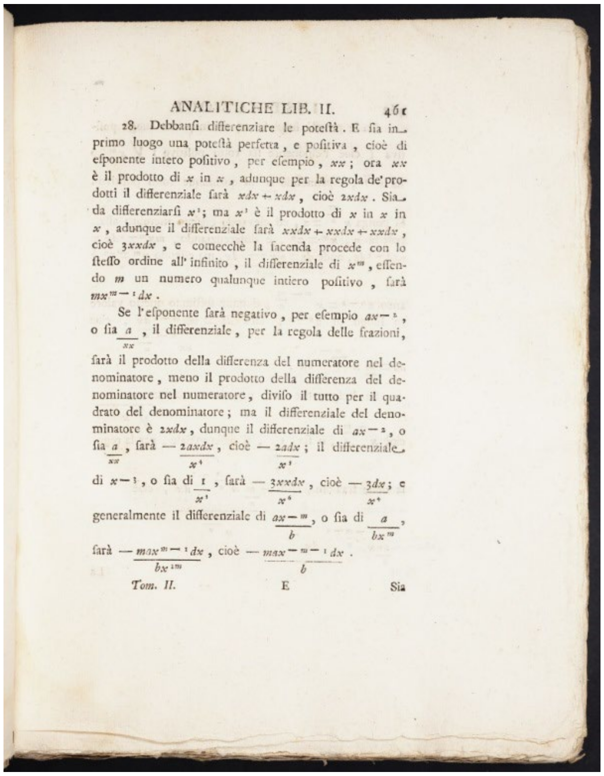
EXAMPLE III.

Fig. 135.

Let the curve to be deſcribed be that of Prob. III. § 238, called the *Witch*, the equation of which is $y = \frac{a\sqrt{ax - xx}}{x}$, the diameter of the circle, being $AC = a$. Take any line at pleaſure, $AB = x$, and draw

You can explore a GeoGebra applet for Witch of Agnesi at https://www.geogebra.org/m/Juvn9xad .

Since the Linda Hall Library collections include both the original Italian version and the English translation, readers are able to take a look at the volumes side by side. In doing so, it is interesting to note that in the original Italian version, Agnesi uses the notation and terminology of Leibniz, while in the English translation, Colson uses Newton's notation and terminology. For an example, see the images below from Section 28 of Volume 2.

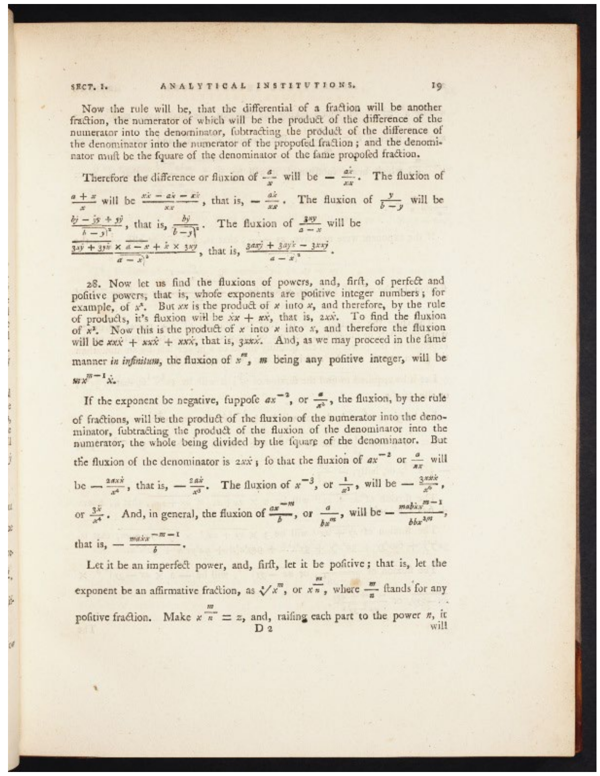


ANALITICHE LIB. II. 461

28. Debbansi differenziare le potestà. E sia in primo luogo una potestà perfetta, e positiva, cioè di esponente intero positivo, per esempio, xx; ora xx è il prodotto di x in x, adunque per la regola de' prodotti il differenziale sarà $xdx + xdx$, cioè $2xdx$. Sia da differenziarsi x^3; ma x^3 è il prodotto di x in x in x, adunque il differenziale sarà $xxdx + xxdx + xxdx$, cioè $3xxdx$, e comecchè la facenda procede con lo stesso ordine all' infinito, il differenziale di x^m, essendo m un numero qualunque intiero positivo, sarà $mx^{m-1}dx$.

Se l'esponente sarà negativo, per esempio ax^{-2}, o sia $\frac{a}{xx}$, il differenziale, per la regola delle frazioni, sarà il prodotto della differenza del numeratore nel denominatore, meno il prodotto della differenza del denominatore nel numeratore, diviso il tutto per il quadrato del denominatore; ma il differenziale del denominatore è $2xdx$, dunque il differenziale di ax^{-2}, o sia $\frac{a}{xx}$, sarà $-\frac{2axdx}{x^4}$, cioè $-\frac{2adx}{x^3}$; il differenziale di x^{-3}, o sia di $\frac{1}{x^3}$, sarà $-\frac{3xxdx}{x^6}$, cioè $-\frac{3dx}{x^4}$; e generalmente il differenziale di $\frac{ax^{-m}}{b}$, o sia di $\frac{a}{bx^m}$, sarà $-\frac{max^{m-1}dx}{bx^{2m}}$, cioè $-\frac{max^{-m-1}dx}{b}$.

Tom. II. E Sia



SECT. I. ANALYTICAL INSTITUTIONS. 19

Now the rule will be, that the differential of a fraction will be another fraction, the numerator of which will be the product of the difference of the numerator into the denominator, subtracting the product of the difference of the denominator into the numerator of the proposed fraction; and the denominator must be the square of the denominator of the same proposed fraction.

Therefore the difference or fluxion of $\frac{a}{x}$ will be $-\frac{a\dot{x}}{xx}$. The fluxion of $\frac{a+x}{x}$ will be $\frac{x\dot{x} - a\dot{x} - x\dot{x}}{xx}$, that is, $-\frac{a\dot{x}}{xx}$. The fluxion of $\frac{y}{b-y}$ will be $\frac{b\dot{y} - y\dot{y} + y\dot{y}}{\overline{b-y}|^2}$, that is, $\frac{b\dot{y}}{\overline{b-y}|^2}$. The fluxion of $\frac{3xy}{a-x}$ will be $\frac{\overline{3x\dot{y} + 3y\dot{x}} \times \overline{a-x} + \dot{x} \times 3xy}{\overline{a-x}|^2}$, that is, $\frac{3ax\dot{y} + 3ay\dot{x} - 3xx\dot{y}}{\overline{a-x}|^2}$.

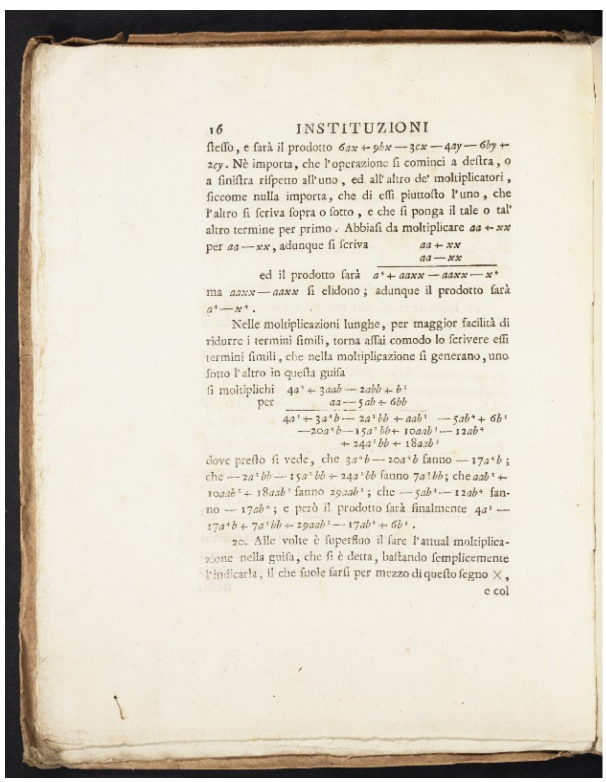
28. Now let us find the fluxions of powers, and, first, of perfect and positive powers, that is, whose exponents are positive integer numbers; for example, of x^2. But xx is the product of x into x, and therefore, by the rule of products, it's fluxion will be $\dot{x}x + x\dot{x}$, that is, $2x\dot{x}$. To find the fluxion of x^3. Now this is the product of x into x into x, and therefore the fluxion will be $xx\dot{x} + xx\dot{x} + xx\dot{x}$, that is, $3xx\dot{x}$. And, as we may proceed in the same manner *in infinitum*, the fluxion of x^m, m being any positive integer, will be $mx^{m-1}\dot{x}$.

If the exponent be negative, suppose ax^{-2}, or $\frac{a}{x^2}$, the fluxion, by the rule of fractions, will be the product of the fluxion of the numerator into the denominator, subtracting the product of the fluxion of the denominator into the numerator, the whole being divided by the square of the denominator. But the fluxion of the denominator is $2x\dot{x}$; so that the fluxion of ax^{-2} or $\frac{a}{xx}$ will be $-\frac{2ax\dot{x}}{x^4}$, that is, $-\frac{2a\dot{x}}{x^3}$. The fluxion of x^{-3}, or $\frac{1}{x^3}$, will be $-\frac{3xx\dot{x}}{x^6}$, or $\frac{3\dot{x}}{x^4}$. And, in general, the fluxion of $\frac{ax^{-m}}{b}$, or $\frac{a}{bx^m}$, will be $-\frac{mab\dot{x}x^{m-1}}{bbx^{2m}}$, that is, $-\frac{ma\dot{x}x^{-m-1}}{b}$.

Let it be an imperfect power, and, first, let it be positive; that is, let the exponent be an affirmative fraction, as $\sqrt[n]{x^m}$, or $x^{\frac{m}{n}}$, where $\frac{m}{n}$ stands for any positive fraction. Make $x^{\frac{m}{n}} = z$, and, raising each part to the power n, it

D 2 will

The picture below from page 16 of volume 1 in Italian shows examples of polynomial multiplication.



16 INSTITUZIONI

stesso, e sarà il prodotto $6ax + 9bx - 3cx - 4ay - 6by + 2cy$. Nè importa, che l'operazione si cominci a destra, o a sinistra rispetto all'uno, ed all'altro de' moltiplicatori, siccome nulla importa, che di essi piuttosto l'uno, che l'altro si scriva sopra o sotto, e che si ponga il tale o tal' altro termine per primo. Abbiasi da moltiplicare $aa + xx$ per $aa - xx$, adunque si scriva

$$\begin{array}{r} aa + xx \\ aa - xx \\ \hline \end{array}$$

ed il prodotto sarà $a^4 + aaxx - aaxx - x^4$

ma $aaxx - aaxx$ si elidono; adunque il prodotto sarà $a^4 - x^4$.

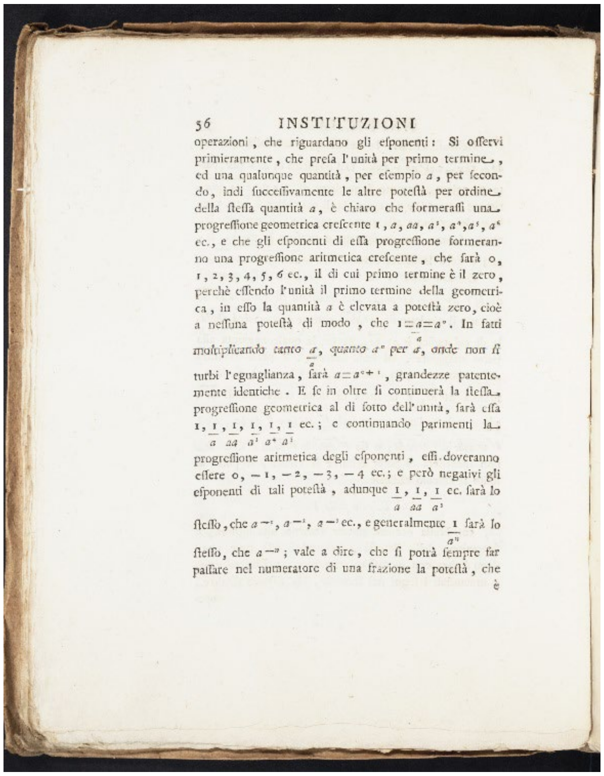
Nelle moltiplicazioni lunghe, per maggior facilità di ridurre i termini simili, torna assai comodo lo scrivere essi termini simili, che nella moltiplicazione si generano, uno sotto l'altro in questa guisa

$$\begin{array}{ll} \text{si moltiplichi} & 4a^3 + 3aab - 2abb + b^3 \\ \text{per} & aa - 5ab + 6bb \\ \hline & 4a^5 + 3a^4b - 2a^3bb + aab^3 - 5ab^4 + 6b^5 \\ & -20a^4b - 15a^3bb + 10aab^3 - 12ab^4 \\ & + 24a^3bb + 18aab^3 \end{array}$$

dove presto si vede, che $3a^4b - 20a^4b$ fanno $-17a^4b$; che $-2a^3bb - 15a^3bb + 24a^3bb$ fanno $7a^3bb$; che $aab^3 + 10aab^3 + 18aab^3$ fanno $29aab^3$; che $-5ab^4 - 12ab^4$ fanno $-17ab^4$; e però il prodotto sarà finalmente $4a^5 - 17a^4b + 7a^3bb + 29aab^3 - 17ab^4 + 6b^5$.

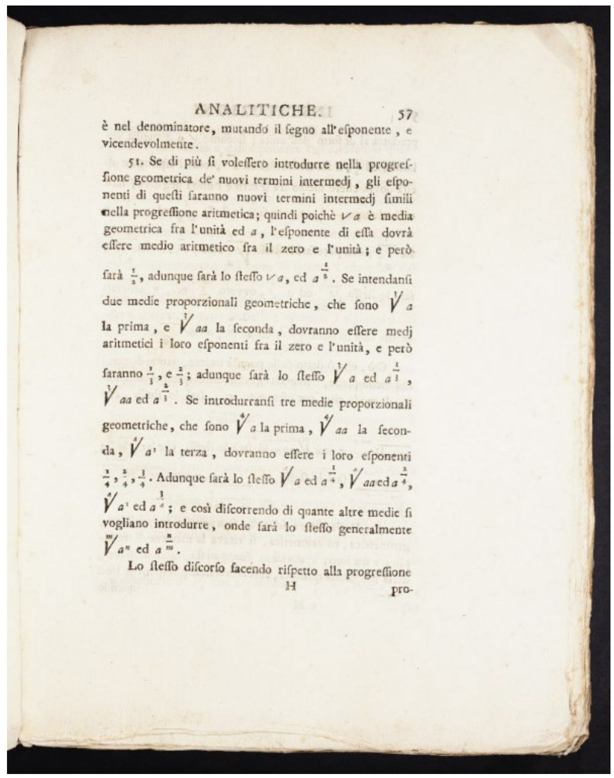
20. Alle volte è superfluo il fare l'attual moltiplicazione nella guisa, che si è detta, bastando semplicemente l'indicarla, il che suole farsi per mezzo di questo segno $\times$,

e col



operazioni, che riguardano gli eſponenti: Si oſſervi primieramente, che preſa l'unità per primo termine, ed una qualunque quantità, per eſempio a, per ſecondo, indi ſucceſſivamente le altre poteſtà per ordine della ſteſſa quantità a, è chiaro che formeraſſi una progreſſione geometrica creſcente $1, a, aa, a^3, a^4, a^5, a^6$ ec., e che gli eſponenti di eſſa progreſſione formeranno una progreſſione aritmetica creſcente, che ſarà $0, 1, 2, 3, 4, 5, 6$ ec., il di cui primo termine è il zero, perchè eſſendo l'unità il primo termine della geometrica, in eſſo la quantità a è elevata a poteſtà zero, cioè a neſſuna poteſtà di modo, che $1 = \frac{a}{a} = a^0$. In fatti moltiplicando tanto $\frac{a}{a}$, quanto a^0 per a, onde non ſi turbi l'eguaglianza, ſarà $a = a^{0+1}$, grandezze patentemente identiche. E ſe in oltre ſi continuerà la ſteſſa progreſſione geometrica al di ſotto dell'unità, ſarà eſſa $1, \frac{1}{a}, \frac{1}{aa}, \frac{1}{a^3}, \frac{1}{a^4}, \frac{1}{a^5}$ ec.; e continuando parimenti la progreſſione aritmetica degli eſponenti, eſſi doveranno eſſere $0, -1, -2, -3, -4$ ec.; e però negativi gli eſponenti di tali poteſtà, adunque $\frac{1}{a}, \frac{1}{aa}, \frac{1}{a^3}$ ec. ſarà lo ſteſſo, che a^{-1}, a^{-2}, a^{-3} ec., e generalmente $\frac{1}{a^n}$ ſarà lo ſteſſo, che a^{-n}; vale a dire, che ſi potrà ſempre far paſſare nel numeratore di una frazione la poteſtà, che è

Negative exponents are explained on page 56 of Volume 1 and



è nel denominatore, mutando il ſegno all'eſponente, e vicendevolmente.

51. Se di più ſi voleſſero introdurre nella progreſſione geometrica de' nuovi termini intermedj, gli eſponenti di queſti ſaranno nuovi termini intermedj ſimili nella progreſſione aritmetica; quindi poichè $\sqrt{a}$ è media geometrica fra l'unità ed a, l'eſponente di eſſa dovrà eſſere medio aritmetico fra il zero e l'unità; e però ſarà $\frac{1}{2}$, adunque ſarà lo ſteſſo $\sqrt{a}$, ed $a^{\frac{1}{2}}$. Se intendanſi due medie proporzionali geometriche, che ſono $\sqrt[3]{a}$ la prima, e $\sqrt[3]{aa}$ la ſeconda, dovranno eſſere medj aritmetici i loro eſponenti fra il zero e l'unità, e però ſaranno $\frac{1}{3}$, e $\frac{2}{3}$; adunque ſarà lo ſteſſo $\sqrt[3]{a}$ ed $a^{\frac{1}{3}}$, $\sqrt[3]{aa}$ ed $a^{\frac{2}{3}}$. Se introdurranſi tre medie proporzionali geometriche, che ſono $\sqrt[4]{a}$ la prima, $\sqrt[4]{aa}$ la ſeconda, $\sqrt[4]{a^3}$ la terza, dovranno eſſere i loro eſponenti $\frac{1}{4}, \frac{2}{4}, \frac{3}{4}$. Adunque ſarà lo ſteſſo $\sqrt[4]{a}$ ed $a^{\frac{1}{4}}$, $\sqrt[4]{aa}$ ed $a^{\frac{2}{4}}$, $\sqrt[4]{a^3}$ ed $a^{\frac{3}{4}}$; e così diſcorrendo di quante altre medie ſi vogliano introdurre, onde ſarà lo ſteſſo generalmente $\sqrt[m]{a^n}$ ed $a^{\frac{n}{m}}$.

Lo ſteſſo diſcorſo facendo riſpetto alla progreſſione

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Among the recognitions that Agnesi received was an appointment to the chair of mathematics at the University of Bologna by Pope Benedict XIV in 1750, two years after the publication of *Instituzioni Analitiche*. The picture below was taken during the 2012 Mathematical Association of America study trip to Italy at the Mathematics Department of the University of Bologna and shows her name included in a list of famous mathematicians connected to the university.

Sources differ, but it appears that Maria did not acknowledge the university appointment. She began to dedicate more of her wealth, time, and energy to charity and religion rather than mathematics, and she completed the transition after her father died in 1752. Agnesi passed away in 1799 in a poorhouse which she had once directed.

 https://www.britannica.com/biography/Maria-Gaetana-Agnesi

By Giovanni Dall'Orto [Attribution], from Wikimedia Commons

[http://e-filatelia.poste.it/showSchedaProdotto.asp?id_prodotto=27800&id_categoria_prodotto=281&id_catalogo_prodotto=2021&lingua]

In order to commemorate of the 300th anniversary of her birth, Italy and the Vatican issued stamps in her honor in 2018.

Required Reading

- **Some Account of Maria Agnesi**, from John Colson's Translation of her *Analytical Institutions*, pages 13 – 17, follow link and scroll down and click on v.1
  https://catalog.lindahall.org/permalink/01LINDAHALL_INST/1nrd31s/alma991877453405961
  The 18th-Century Lady Mathematician Who Loved Calculus and God,
  https://www.smithsonianmag.com/science-nature/18th-century-lady-mathematician-who-changed-how-calculus-was-taught-180969078

Optional Reading and Viewing

- **Maria Agnesi with the Preacher, the Witch, and Calculus –** presentation by Dr. Huffman, https://youtu.be/A4D-zAGRGkc
- **Maria Gaetana Agnesi**, https://en.wikipedia.org/wiki/Maria_Gaetana_Agnesi
- **The World of Maria Gaetana Agnesi, Mathematician of God**, by Massimo Mazzotti, Johns Hopkins, 2007, ISBN 978-0-8018-8709-3
- **Math Equals: Biographies of Women Mathematicians+Related Activities**, by Teri Perl, Addison Wesley, ISBN 0-201-05709-3, p. 52-61.
- **Maria Gaëtana Agnesi,** https://mathshistory.st-andrews.ac.uk/Biographies/Agnesi/